THE NUMBER OF POSSIBLE COLOUR BAND COMBINATIONS FOR USE IN BIRD BANDING OR MARKING.

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ABSTRACT

The various kinds of colour-band lists are discussed and formulae and tables giving the total number of combinations for each kind are given.

INTRODUCTION

The marking of individuals for subsequent recognition is an important technique in population ecology.

Rowley (1964) has distinguished two methods of using colour bands. His "extensive method" applies to the use of a single colour to denote either year classes or regional groups or filial relationships. With this method large numbers can be banded but the amount of information gained is limited.

His "intensive method" is where each banded individual is given a unique colour code. Banded individuals, therefore, can be recognised without recapture. This method enables investigation of many features of the birds' ecology and behaviour. The intensive method is discussed in more detail in the present paper because of the many problems that are encountered when using it.

Errors can occur at three stages in a colour-banding programme. These stages are (1) making up the initial colour-combination list, (2) applying the bands, and (3) recording field observations of banded birds. The first source of error is due to the complexity of list generation. Hand methods are extremely laborious and are very prone to duplication of combinations. In one study known to the author, 3.5 percent of all combinations were duplicates - this resulted in many birds carrying the same colour combination, and, because they could not be distinguished, all these records were useless. Furthermore, many hand generated lists are deficient in that a great number of possible combinations are missed. In one list, of 370 260 possible combinations, only 40 764 were thought to exist - 89 percent of the possible combinations were missed. The second and third sources of error are more easily overcome so only the errors in to generating lists will be considered here.

COLOUR-BANDING TERMINOLOGY

The terminology to be introduced in the following section needs some explanation.

Metal band. These usually carry a unique number and an address for the return of the band. There is usually only one metal band on each bird. The metal band may be a part of the colour combination, in which case it is treated as a colour but is present only once in every combination, or it may be separated from the colours and placed, for example, on the leg which does not carry the colour bands.

Partitioning. This is where a combination is split in all possible ways between the two legs of the bird. For instance, the combination BLUE-METAL-BLACK may be "partitioned" to yield the following four combinations.

	Left leg	Right leg
1.	BLUE-METAL-BLACK	
2.	BLUE-METAL	BLACK
3.	BLUE	METAL-BLACK
4.		BLUE-METAL-BLACK

The advantage of this is that more combinations are obtainable but its disadvantage is that field identification is more difficult because the observer must identify the bird's left and right leg. It is advisable to follow Erickson's (1969) convention regarding the recording of combinations. This is that the order of recording bands is from proximal to distal and from left to right. Erickson's example makes this clear: a bird carrying blue on its left tarsus and metal over red on its right is recorded as B/MR, the slash indicating that the colours to the left are carried on the left tarsus while those to the right are on the right tarsus.

Excluding Adjacent Repeats. This term refers to the process whereby all those combinations having the same coloured bands immediately adjacent to each other on the same leg are excluded from the master list. For example, RR/MW would be excluded because some observers may mistake the solid block of red on the left tarsus as indicating only one band and so record it as R/MW. Combinations such as R/RMW are not excluded because the repeated colours are not adjacent on the same leg. The New Zealand Banding Office strongly recommends that local workers should use lists in which adjacent repeats have been excluded.

CODING METHODS

The coding methods given below cover all the common usages - other codes have been devised but they are generally derivatives of one or other of the basic codes given here. Table 1 gives a summary of the features of each method.

Method 1

This is the simplest method in that there is no partitioning, the aluminium band is not treated as a colour, and adjacent repeats are not excluded. Rowley (1965) coded celluloid colour-bands using this method. The number of combinations possible (P) using this method is:

$P = n^r$ (Buckley and Hancock 1968)

where r bands are to be used on each bird and n is the number of colours available. This method is not only applicable to colour-banding but is also suitable for tagging and painting. Refer to Table 2.

Method 2

This method is the same as Method 1 but adjacent repeats are excluded. The number of combinations is given by:

$$P = n(n-1)^{r-1}$$

Like Method 1 this is a general method. Refer to Table 2.

Table 1. Summary of the features for the coding methods mentioned in the text.

	Metal treated as a colour	Exclude adjacent repeats	Partition	Restrict the no. of bands on a leg	Combination given in Table
Method 1	NO	NO	NO	NO	2
Method 2	NO	YES	NO	NO	2
Method 3	YES	NO	NO	NO	3
Method 4	YES	YES	NO	NO	3
Method 5	NO	NO	YES	NO	4
Method 6	NO	YES	YES	NO	4
Method 7	YES	NO	YES	NO	5
Method 8	YES	YES	YES	NO	5
Method 9	NO	NO	YES	YES	4
Method 10	NO	YES	YES	YES	4
Method 11	YES	NO	YES	YES	5
Method 12	YES	YES	YES	YES	5

Method 3

The colour combinations of r bands are drawn from n colours including a metal band being treated as a colour, but without partitioning and without excluding adjacent repeats. This coding method has been used by Ellis (1960). The number of combinations is given by:

$P = (r+1)n^r$ (Buckley and Hancock 1968)

Computer programs to generate lists based on this method are given by Buckley and Hancock (1968) and Duncan (1972). The former program is in FORTRAN-II while the later is in FORTRAN-IV. This method is specific to colour-banding. Refer to Table 3.

Method 4

This is the same as Method 3 but excludes adjacent repeats. The number of combinations is given by:

$P = (r+1)n(n-1)^{r-1}$

A computer program to generate lists based on this is given by Duncan (1972). This method is specific to colour-banding. Refer to Table 3.

Method 5

In this method colour combinations of r colour bands are drawn from n colours without a metal band being treated as a colour, but with partitioning. This is the same as Method 1 except for partitioning. The number of combinations is given by:

$P = (r+2)n^r$

This method is suitable for colour-banding or painting where paint is applied to both the tail and wing feathers. Refer to Table 4.

Method 6

This method is the same as Method 5 except that it excludes adjacent repeats. No simple formula is known for the number of combinations but see Table 4 for solutions. The applicability is the same as for Method 5.

Method 7

In this method the colour combinations treat the metal band as a colour and there is partitioning but adjacent repeats are not excluded. This method is the same as Method 3 but with partitioning. The number of combinations is given by:

$P = (r+2)\ (r+1)n^r$

This method is specific to colour-banding. Refer to Table 5.

Method 8

This method is the same as Method 7 except for excluding adjacent repeats. No simple formula is known for the number of combinations but see Table 5 for the Monte Carlo solutions. This method is specific to colour-banding.

Method 9

The colour combinations do not use the metal band as a colour or exclude adjacent repeats but there is partitioning and the number of bands per leg is restricted to s. The number of combinations is given by:

$P = (2s-r)n^r$

This is suitable for both colour-banding and painting. Refer to Table 4.

Method 10

This method is the same as Method 9 except for excluding adjacent repeats. No simple formula is known for the number of combinations but see Table 4 for Monte Carlo solutions. The applicability is the same as for Method 9.

Method 11

The metal ban is treated as a colour, and there is partitioning and the number of bands per leg is restricted to s. Erickson (1969) provides an example for the use of this method. The number of combinations is given by:

$$P = (2s-r)\ (r+1)n^r$$

This method is specific to colour-banding. Refer to Table 5.

Method 12

This method is the same as Method 11 except for excluding adjacent repeats. No simple formula is know for the number of combinations but see Table 5 for Monte Carlo solutions. This method is specific to colour-banding.

DISCUSSION

The ideal coding method is one that uses only a limited number of colours and bands per bird yet allows a large number of combinations to be generated. It should be easy to apply and simple for field observers to decode. It should also allow missing bands to be detected. Obviously no method will satisfy all of these criteria so the one that is adopted must be a compromise. Limiting the number of colours used simplifies coding and can obviate fading problems, but it reduces the number of combinations possible. Limiting the number of bands used per bird makes application and recognition easier, but this too reduces the number of combinations. Detecting missing bands is essential in any study. This can be most readily achieved by putting the same number of bands on every bird (Ellis 1960) but, in large scale studies, the banders may have to increase dramatically the number of different colours or use a large number of bands on each bird in order to provide enough combinations.

Methods which result in a limited number of bands per leg and per bird are to be preferred, as are those which exclude adjacent repeats. Therefore Method 12 is probably best but making up lists of combinations based on this method is very difficult and laborious. Full program listings (in FORTRAN-IV) for any of the methods given here may be obtained from the author.

USE OF THE TABLES

Tables 2 to 5 give the total number of colour combinations for any of the coding methods considered above. The following symbols are used in the tables:

- n - the number of colours;
- r - the number of colour-bands used per leg;
- s - the number of bands (including metal if used as a colour) used per leg.

Neither n nor r include the metal band.

The totals given in the tables can be doubled when banding species with a marked sexual dimorphism.

Table 2 gives the number of combinations for Method 1 (non-underlined entries). If the bands are placed on only one leg for each bird, the number of combinations is double that shown because all the combinations can be placed on the left leg for one group of birds and on the right for another group.

As an example, the number of combinations using three or less bands per bird, with nine colours available and excluding adjacent repeats (Method 2), is 9 + 72 + 576 = 657 (or double if all the bands are placed on one leg). No metal bands are included in these combinations.

Table 3 gives the number of combinations for Method 3 (non-underlined entries) and 4 (underlined entries). Again, the number of entries can be doubled if the bands for one combination are on each leg. As an example the number of combinations using three bands per bird (including metal) with 10 colours available and excluding adjacent repeats (Method 4), is 270 (from $\underline{r}$ = 2, $\underline{n}$ = 10), or double this if all the combinations are used twice by being placed on different legs for different groups.

Table 4 gives the number of combinations for Methods 5 and 9 (non-underlined entries) and 6 and 10 (underlined entries). The full table applies only to Methods 9 and 10. Methods 5 and 6 use only a few of the columns in the table because these methods do not place any restrictions on the number of bands per leg unlike Methods 9 and 10. The columns to be used for Methods 5 and 6 are boxed. For methods 9 and 10 use all of the columns in the table. As an example the number of combinations using three colour bands per bird, from a set of seven colours, and with no more than three bands per leg, is 14 + 147 + 1 372 + 7 203 + 33 614 = 42 350.

Table 5 is for Methods 7 and 11 (non-underlined entries) and 8 and 12 (underlined entries). The full table applies only to Methods 11 and 12. Methods 7 and 8 use only a few columns in the table because these methods do not place any restrictions on the number of bands per leg. The columns to be used for Methods 7 and 8 are boxed. For methods 11 and 12 use all of the columns in the table.

EXAMPLES

Problem 1. What is the total number of combinations possible using five colours taken three at a time, with partitioning, with a metal band being treated as a colour, and excluding adjacent repeats (Method 8).

Number = 1 930 (from $\underline{n}$ = 5, $\underline{r}$ = 3, and using the boxed column).

This list will include some combinations giving four bands on one leg (three colour and one metal) and none on the other.

Problem 2. From a set of five colours it was desired to have as many combinations as possible with no more than three bands (including metal) per leg. The combinations are to include adjacent repeats (Method 11).
The number is 30 (from $\underline{n}$ = 5, $\underline{s}$ = 2) + 300 (from $\underline{n}$ = 5, $\underline{r}$ = 2, $\underline{s}$ = 3) + 1 500 (from $\underline{n}$ = 5, $\underline{r}$ = 3, $\underline{s}$ = 3) + 6 250 (from $\underline{n}$ = 5, $\underline{r}$ = 4, $\underline{s}$ = 3) + 18 750 (from $\underline{n}$ = 5, $\underline{r}$ = 5, $\underline{s}$ = 3) = 26 730. This list will not contain any combination having more than three bands per leg. However, the majority of the combinations entail banding both legs with three bands.

These tables apply to many other marking techniques. Table 2, for example, gives the number of combinations possible when toe-clipping, feather imping, or painting, and Table 3 can be applied to certain painting techniques.

LITERATURE CITED

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Table 2. The number of combinations of r colour bands per bird from a set of n colours (Method 1 non underlined entries and Method 2, underlined entries). See text for explanation.

Numbers of colours used (n)	Number of bands per bird (r)				
	1	2	3	4	5
1	1				
2	2	4			
		2			
3	3	9	27		
		6	12		
4	4	16	64	256	
		12	36	108	
5	5	25	125	625	3125
		20	80	320	1280
6	6	36	216	1296	7776
		30	150	750	3750
7	7	49	343	2401	16807
		42	252	1515	9072
8	8	64	512	4096	32768
		56	392	2744	19208
9	9	81	729	6561	59049
		72	576	4608	36864
10	10	100	1000	10000	100000
		90	810		
11	11	121	1331		
		110	1100		
12	12	144	1728		
		132	1452		
13	13	169	2197		
		156	1872		
14	14	196	2744		
		182	2366		
15	15	225	3375		
		210	2940		
16	16	256	4096		
		240	3600		
17	17	289	4913		
		272	4352		
18	18	324	5832		
		306	5202		
19	19	361	6859		
		342	6156		
20	20	400	8000		
		380	7220		

Table 3. The number of combinations of r bands from a set of n colours with a metal band treated as a colour. (Method 3, non underlined entries and Method 4, underlined entries.) See text for explanation.

Number of colours used (n)	Number of bands per bird (r)				
	1	2	3	4	5
1	2				
2	4	12 6			
3	6	27 18	108 48		
4	8	48 36	256 144	1280 540	
5	10	75 60	500 320	3125 1600	18750 7680
6	12	108 90	864 600	6840 3750	46656 22500
7	14	146 126	1372 1008	12005 7560	100842 54432
8	16	192 168	2048 1568		
9	18	243 216	2916 2304		
10	20	300 270	4000 3240		
11	22	363 330	5324 4400		
12	24	432 396	6912 5808		
13	26	507 468	8788 7488		
14	28	588 546	10976 9464		
15	30	675 630			
16	32	768 720			
17	34	867 816			
18	36	972 918			
19	38	1083 1026			
20	40	1200 1140			

Table 4. The number of combinations of r colour bands per bird drawn from n colours in which the metal band is not included in the combination but where the colour sequences are partitioned (Methods 5, 6, 9, and 10). For Method 5, use the non underlined values in the boxed columns. For Method 6, use the underlined values in the boxed columns. For Method 9, use the non underlined values in the whole table. For Method 10, use the underlined values in the whole table. See text for explanation.

	Number of colours used per bird (r)														
	1	2		3		4			5						
Number of colours used (n)	Number of bands per leg (s)	Number of bands per leg (s)		Number of bands per leg (s)		Number of bands per leg (s)			Number of bands per leg (s)						
	1	1	2	2	3	2	3	4	3	4	5				
1	2														
	2														
2	4	4	12												
	4	4	8												
3	6	9	27	54	108										
	6	9	21	36	60										
4	8	16	48	128	256	256	768	1280							
	8	16	40	96	168	144	432	648							
5	10	25	75	250	500	625	1875	3125	6250	12500	18750				
	10	25	65	200	360	400	1200	1840	3200	6400	8960				
6	12	36	108	432	864	1296	3888	6480	15552	31104	46656				
	12	36	96	360	660	900	2700	4200	9000	18000	25500				
7	14	49	147	686	1372	2401	7203	12005	33614	67228	100842				
	14	49	133	583	1092	1764	5292	8316	21168	42336	60480				
8	16	64	192	1024	2048	4096	12288								
	16	64	176	896	1680	3136	9408								
9	18	81	243	1458	2916	6561									
	18	81	225	1296	2448	5184									
10	20	100	300	2000	4000	10000									
	20	100	280	1800	3420	8100									

Table 4.(continued)

	Number of colours used per bird (r)									
	1	2		3		4			5	
Number of colours used (n)	Number of bands per leg (s)	Number of bands per leg (s)		Number of bands per leg (s)		Number of bands per leg (s)			Number of bands per leg (s)	
	1	1	2	2	3	2	3	4	3	4	5
11	22	121	363	2662	5324						
	22	121	341	2420	4620						
12	24	144	432	3456	6912						
	24	144	408	3168	6072						
13	26	169	507	4394	8788						
	26	169	481	4056	7800						
14	28	196	588	5488							
	28	196	560	5096							
15	30	225	675	6750							
	30	225	645	6300							
16	32	256	768	8192							
	32	256	736	7680							
17	34	289	867								
	34	289	833								
18	36	324	972								
	36	324	936								
19	38	361	1083								
	38	361	1045								
20	40	400	1200								
	40	400	1160								

Table 5. The number of combinations of r colour bands per bird from n colours with a metal band included in each sequence. The metal band is treated as a colour and the sequences of colours are partitioned (Method 7, 8, 11, and 12). For Method 7 use the non underlined values in the boxed columns. For Method 8 use the underlined entries in the boxed columns. For Method 11 use the non underlined entries in the whole table. For Method 12 use the underlined entries in the whole table. See text for explanation.

	Number of colours used per bird (r)														
	1		2		3			4			5				
Number of colours used (n)	Number of bands per leg (s)										Number of bands per leg (s)				
	1	2	2	3	2	3	4	3	4	5	3	4	5	6	
1	2	6													
	2	6													
2	4	12	24	48											
	4	12	20	36											
3	6	18	54	108	108	324	540								
	6	18	48	90	72	222	342								
4	8	24	96	192	256	768	1280	2560	5120	7680					
	8	24	88	168	192	584	920	1536	3096	4392					
5	10	30	150	300	500	1500	2500	6250	12500	18750	18750	56250	93750	131250	
	10	30	140	270	400	1210	1930	4200	8440	12120	10400	31200	52160	70080	
6	12	36	216	432	864	2592	4320	12960	25920	38880	46656	139968	233328	326592	
	12	36	204	396	720	2172	3492	9360	18780	27180	28800	86400	144300	195300	
7	14	42	294	588	1372	4116	6860	24010	48020	72030	100842	302526	504210	705894	
	14	42	280	546	1176	3542	5726	18228	36540	53172	67032	201096	335664	456624	
8	16	48	384	768	2048	6144									
	16	48	368	720	1792	5392									
9	18	54	486	972	2916	8748									
	18	54	468	918	2592	7794									
10	20	60	600	1200	4000										
	20	60	580	1140	3600										

Table 5. (continued)

	Number of colours used per bird (r)														
	1		2		3			4			5				
Number of colours used (n)	Number of bands per leg (s)		Number of bands per leg (s)		Number of bands per leg (s)			Number of bands per leg (s)			Number of bands per leg (s)				
	1	2	2	3	2	3	4	3	4	5	3	4	5	6	
11	22	66	726	1452	5324										
	22	66	704	1386	4840										
12	24	72	864	1728	6912										
	24	72	840	1656	6336										
13	26	78	1014	2028	8788										
	26	78	988	1950	8112										
14	28	84	1176	2352											
	28	84	1148	2268											
15	30	90	1350	2700											
	30	90	1320	2610											
16	32	96	1536	3072											
	32	96	1504	2976											
17	34	102	1734	3468											
	34	102	1700	3366											
18	36	108	1944	3888											
	36	108	1908	3780											
19	38	114	2166	4332											
	38	114	2128	4218											
20	40	120	2400	4800											
	40	120	2360	4680											